Application of analogy of momentum and heat transfer at shell and tube condenser

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In environmental and chemical industry majority of processes are based on the simultaneous transfer of extensive properties. These can be the heat, mass and momentum transfer. Because of the close relation of these transport phenomena, correlations can be created between transport coefficients. These are the so-called transport analogies. In this paper relation of heat and momentum transfer is studied. In order to perform the analysis, experimental investigations were made on a shell and tube heat exchanger device. Goals of our work are to calculate tube side heat transfer coefficient and Nu number with formulas of transport analogies; to validate them by measured data; to find reason of decreasing Nu number with increasing Re number in the condenser at higher Re levels. Finally operation of the condenser is modelled by the analogy of heat and momentum transfer.

\textit{Keywords:} shell and tube heat exchanger, transport analogy, friction factor, heat transfer coefficient

1. Introduction

In environmental and chemical engineering majority of processes are based on the simultaneous transfer of extensive properties. These can be the heat-, mass- and momentum transfer. Because of the close relation of these transport phenomena, correlations can be created among transport properties.

At the end of 19\textsuperscript{th} century Osborne Reynolds was the first who recognized the analogy between heat and momentum transfer during his experimental investigation on rough tubes. His observations resulted in the so-called Reynolds analogy which describes the connection between heat transfer coefficient and friction factor [1, 2]:

\[ St = \frac{Nu}{Re \cdot Pr} = \frac{h}{\rho c_p \nu} = \frac{f_e}{2} \quad (1) \]

Eq (1) is applicable for turbulent flow in a pipe or over a flat plate when Pr is equal or close to unity. This is a reasonable approximation for many gases, but for liquids the Pr numbers are much larger than 1. Therefore this analogy is not appropriate for liquids.

Since applicability of the Reynolds analogy is very limited, derivation of a more general expression was necessary. In independent works Ludwig Prandtl and Geoffrey Ingram Taylor modified the Reynolds analogy in the first half of the 20\textsupersth century. A new formula was created between heat transfer coefficient and friction factor by combining the resistances of a viscous sublayer where molecular effects dominate with a turbulent outer layer where turbulent effects control [1, 2]:

\[ St = \frac{h}{\rho c_p \nu} = \frac{f_e}{2} \quad (2) \]

Prandtl-Taylor analogy can be applied in a wider range, it allows Pr to differ from 1. Hence it is applicable mainly in case of Pr smaller than 2 [1].

Theodore von Kármán (1881-1963) extended Reynolds analogy even further by including an intermittent boundary layer between the viscous sublayer and the turbulent bulk. Turbulent viscosity and turbulent heat conductivity were determined from the universal velocity profile. Thus the Kármán analogy is [1, 3]:
Thomas H. Chilton and Allan P. Colburn expanded the Prandtl-Taylor analogy for fluids with varying Prandtl number. Intensive properties (velocity and temperature) were assumed to be varying linearly in the intermittent boundary layer similarly to the viscous sublayer [1]. Chilton-Colburn analogy (1933) – also called j factor analogy – is also a modification of the Reynolds analogy based on purely experimental results [4]. Regarding to heat and momentum transfer [1]:

\[
m = \frac{j}{\mu} \cdot \frac{f_d}{2} \cdot \frac{1}{Pr} 
\]

Where \(z\) is an experimentally determined constant, e.g., at fluid flows inside tubes \(z = 2/3\). Above mentioned j factor in Eq(4) can be determined with friction factor only, if drag coefficient comes from friction and shape resistance can be neglected (e.g., flows inside or outside tube parallel to its longitudinal axis, flows above a flat plate) [1]. Chilton-Colburn analogy is still popular and widely used because of its accuracy and simplicity. For instance, it can be applied reasonably for industrial sugarcane juices in shell and tube heat exchangers [5], and at shape optimization of chevron-type plate heat exchangers [6].

At fluids characterized by small \(Pr\) numbers (e.g., liquid phase metals) the Martinelly analogy can be applied. At this analogy Nu-Re-Pr correlation is represented by monograms.

In the middle of 20th century Robert Dießler proved that at increasing \(Pr\) numbers (\(Pr >> 10\)) result of the Kármán analogy differ more and more from the experimentally obtained data. It was recognized that near to the wall where high temperature gradients exist description of the eddy diffusivity had to be modified. Result of Dießler analogy is [3]:

\[
St = 0,111 \cdot \frac{f_d}{2} \cdot Pr^{-3/4} 
\]

W. L. Friend and A. D. Metzner made measurements in a wide range of Sc and Pr numbers to obtain a correlation between heat-, mass- and momentum transfer [4]:

\[
St = \frac{f_d \cdot 0,14}{2 \cdot Pr^{-1/3}} 
\]

Although Eq (6) is recommended for wide range of \(Pr\) numbers, at small ones (\(Pr < 0.5\)) it can not be applied.

2. Material and methods

Test device

Experimental measurements were conducted on a shell and tube heat exchanger system (Table 1, Fig. 1). The first unit of the system is a condenser with moderate pressure steam in the shell side and water as the product in tube side. The second unit is a cooler where the heated product coming from the first body is cooled down before letting it to the drainage system. This work is focused on the process taking place in the condenser. The targeted device is two pass on tube side and one pass on shell side where the saturated steam coming from an electric steam generator is admitted. The temperature of steam is measured and kept constant (\(\approx 103^\circ C\)) during the measurement process. Mass flow rate of steam is measured by measuring weighing the mass of condensate after the steam trap by using a stop watch and a bucket. Flow rate of product is measured by a rotameter. Inlet and outlet temperatures are measured by temperature transmitters (Fe-CuNi thermocouples), temperature data are collected by an ALMEMO data collector and recorded by Win Control. During the experimental work product flow rate isvaried between 600 and 1300l/h. At different product flow rates - after reaching the steady state operating mode - temperature and flow rate data are recorded. At each individual operating state heat balance and heat transfer coefficients are evaluated.

Table 1. Data of the shell and tube heat exchanger device

<table>
<thead>
<tr>
<th>Heat transfer area</th>
<th>Number of tubes</th>
<th>Size of tubes</th>
<th>Size of shell</th>
<th>Heat conductivity of the wall</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A = 1.5\ m^2)</td>
<td>(N = 48)</td>
<td>(L = 1\ m;)</td>
<td>(L = 1\ m;)</td>
<td>(k_w = 14\ W/mK)</td>
</tr>
<tr>
<td>(d_{in}/d_o = 10/8\ mm)</td>
<td>(d_{in}/d_o = 200/196\ mm)</td>
<td></td>
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</tbody>
</table>
Evaluation of heat transfer coefficient

There is sensible heat transfer in tube side, while condensation on shell side in the first unit of the device. The tube side heat transfer coefficient can be determined from the next equation:

$$h = \frac{1}{\frac{1}{\overline{v}} - \frac{1}{h_{cond}} - \left(\frac{2}{k_w}\right)}$$

(7)

The condensation heat transfer coefficient along vertical tubs can be calculated by the next correlation [7]:

$$h_{cond} = 1.47 \cdot R_{e_f}^{-1/3} \cdot \left[k_f \cdot \rho_f \cdot \beta \cdot \mu_f^2 \right]^{1/3}$$

(8)

In Eq (8) the Reynolds number of the condensation film:

$$R_{e_f} = \frac{4 \cdot m_{st}}{N \cdot d_{out} \cdot \pi \cdot \mu_f}$$

(9)

The overall heat transfer coefficient can be calculated from the transferred heat flow rate and the logarithmic temperature difference:

$$U = \frac{\dot{Q}_{tr}}{\Delta T_{in} \cdot A}$$

(10)

The transferred heat is the average of deflated and admitted heat:

$$\dot{Q}_{tr} = \frac{\dot{Q}_{defl} \cdot \dot{Q}_{adm}}{2}$$

(11)

Deflated heat flow rate by the steam on shell side:

$$\dot{Q}_{defl} = m_{st} \cdot \Delta h_{st}$$

(12)

Admitted heat flow rate by the product on tube side [9, 10]:

$$\dot{Q}_{adm} = m_{pr} \cdot c_{pr} \cdot (T_{out} - T_{in})$$

(13)

During the conducted series of experimental measurements it was found that - contrary to expectations - the tube side heat transfer coefficient (and Nu number) at a distinct level of Re number does not continue increasing but slightly decreases instead (at $T_{st} \approx 103^\circ C$, this level is $Re \approx 4000$) (Fig. 2). Just before this level of Re number temperature of the condensate measured at the outlet pipe of the shell side starts strongly falling (Fig. 3). It can be suspected that condensate does not leave the device due to inappropriate installation and shell side becomes flooded (Fig. 4). This phenomenon is realized directly through the cut down of the condensate temperature. To verify this hypothesis we turned to transport analogies to simulate the operation of the condenser.

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**Fig. 1. Flow chart of the test equipment**
3. Results

Since all analogies are very sensible to friction factor, exact determination of that has high relevance. Several friction models were investigated (Hagen-Poiseuille, Blasius, Kármán, Colebrook-White, Panhandle-A) but two of them can be considered due to roughness of the inside pipe wall and Re restrictions: Blasius formula can be applied in the turbulent range (2300...3000 < Re < 10^5) for hydraulic smooth tubes:

$$f_d = \frac{0.03164}{Re^{1/4}}$$

and Panhandle-A formula [8] is useful at the lower part of turbulent range for smooth tubes:

$$f_d = \frac{4}{47.22 \cdot E^{2} \cdot Re^{0.146}}$$

E is the efficiency coefficient which can vary from 0.85 to 1. In Eq (14) and Eq (15) express Darcy’s friction which is in direct connection with Fanning friction applied in analogies’ equations Eq(1)...Eq(6):

$$f_e = f_d$$

Application of analogies was investigated by looking for the minimum of sum of squared residuals:

$$SSR = \sum_{i=1}^{n} (Nu_{meas} - Nu_{pred}(Re))$$

All analogies were combined with different friction correlations (Blasius, Panhandle-A with E = 0.85, 0.9, 0.95, 1.0). SSR values can be found in (Table 2). It can be seen that measured Nu numbers do not fit on Reynolds analogy and Diessler analogy at all. Prandtl-Taylor, Kármán, Chilton-Colburn and Friend Metzner analogy can be applied quite well with friction given by Panhandle-A formula, but friction given by Blasius formula can be applied only with Friend-Metzner analogy. The minimum value of SSR is obtained by Chilton-Colburn analogy with friction by Panhandle-A with E = 0.9. Result of this analogy with measured values can be seen in Fig. 5.
At Re > 4000 the measured heat transfer coefficients and Nu numbers start falling because of suspected reduction of condensation heat transfer area of the device which is probably caused by condensate retention. Due to inadequate condensate drainage in the bottom of the heat exchanger there is sensible heat transfer instead of condensation (Fig. 4). At Re > 4000 modified heat transfer area can be calculated from the predicted heat transfer coefficient from the Chilton-Colburn analogy by Eq (18) and Eq (19). The predicted condensate level is represented in Fig. 6.

\[ U' = \frac{1}{h_C - c} + \frac{1}{h_{cond}} + \left(\frac{c}{k}\right)^\gamma \]  
\[ A' = \frac{\dot{Q}_{tr}}{\Delta T_{in} \cdot U'\prime} \]  

4. Conclusion

Several analogies of heat and momentum transfer can be applied quite well at shell and tube heat exchanger. Since all analogies are very sensible on friction coefficient, determination of its exact value needs further investigations. Inappropriate operation of the condenser could be modeled successfully. Installation of a condensate pump in the system is recommended to prevent flooding of the shell (Fig.6).
APPLICATION OF ANALOGY OF MOMENTUM AND HEAT TRANSFER AT SHELL AND TUBE CONDENSER

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
<th>Subscripts</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>surface</td>
<td>m²</td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>specific heat</td>
<td>J/kgK</td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>diameter</td>
<td>m</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>hydraulic efficiency coefficient</td>
<td>[1]</td>
<td></td>
</tr>
<tr>
<td>f</td>
<td>friction factor</td>
<td>[1]</td>
<td></td>
</tr>
<tr>
<td>g</td>
<td>gravitational acceleration</td>
<td>[m/s²]</td>
<td></td>
</tr>
<tr>
<td>h</td>
<td>heat transfer coefficient</td>
<td>[W/m²K]</td>
<td></td>
</tr>
<tr>
<td>δh</td>
<td>heat of condensation</td>
<td>[J/kg]</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>height</td>
<td>[m]</td>
<td></td>
</tr>
<tr>
<td>j</td>
<td>colburn factor</td>
<td>[1]</td>
<td></td>
</tr>
<tr>
<td>k</td>
<td>heat conductivity</td>
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<tr>
<td>L</td>
<td>length</td>
<td>[m]</td>
<td></td>
</tr>
<tr>
<td>m</td>
<td>mass flow rate</td>
<td>[kg/s]</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>number of tubes</td>
<td>[1]</td>
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</tr>
<tr>
<td>Nu</td>
<td>Nusselt number</td>
<td>[1]</td>
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</tr>
<tr>
<td>Pr</td>
<td>Prandtl number</td>
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<tr>
<td>Qt</td>
<td>heat flow rate</td>
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<td>Re</td>
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</tr>
<tr>
<td>S</td>
<td>wall thickness</td>
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</tr>
<tr>
<td>SSR</td>
<td>sum of squared residuals</td>
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</tr>
<tr>
<td>St</td>
<td>Stanton number</td>
<td>[1]</td>
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</tr>
<tr>
<td>T</td>
<td>temperature</td>
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</tr>
<tr>
<td>ΔT</td>
<td>temperature difference</td>
<td>[°C]</td>
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<tr>
<td>U</td>
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<td>z</td>
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<tr>
<td>μ</td>
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<tr>
<td>v</td>
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<tr>
<td>ρ</td>
<td>density</td>
<td>[kg/m³]</td>
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</tbody>
</table>

Subscripts:

adm: admitted

Ch-C: Chilton-Colburn

cond: condensation

D: Darcy's

defl: deflated

F: Fanning

f: condensate film

H: heat transfer

in: inside/inlet

ln: logarithmic

meas: measured

out: outside/outlet

p: at constant pressure

pr: product

pred: predicted

s: steam

sens: sensible

tr: transferred

w: wall

References